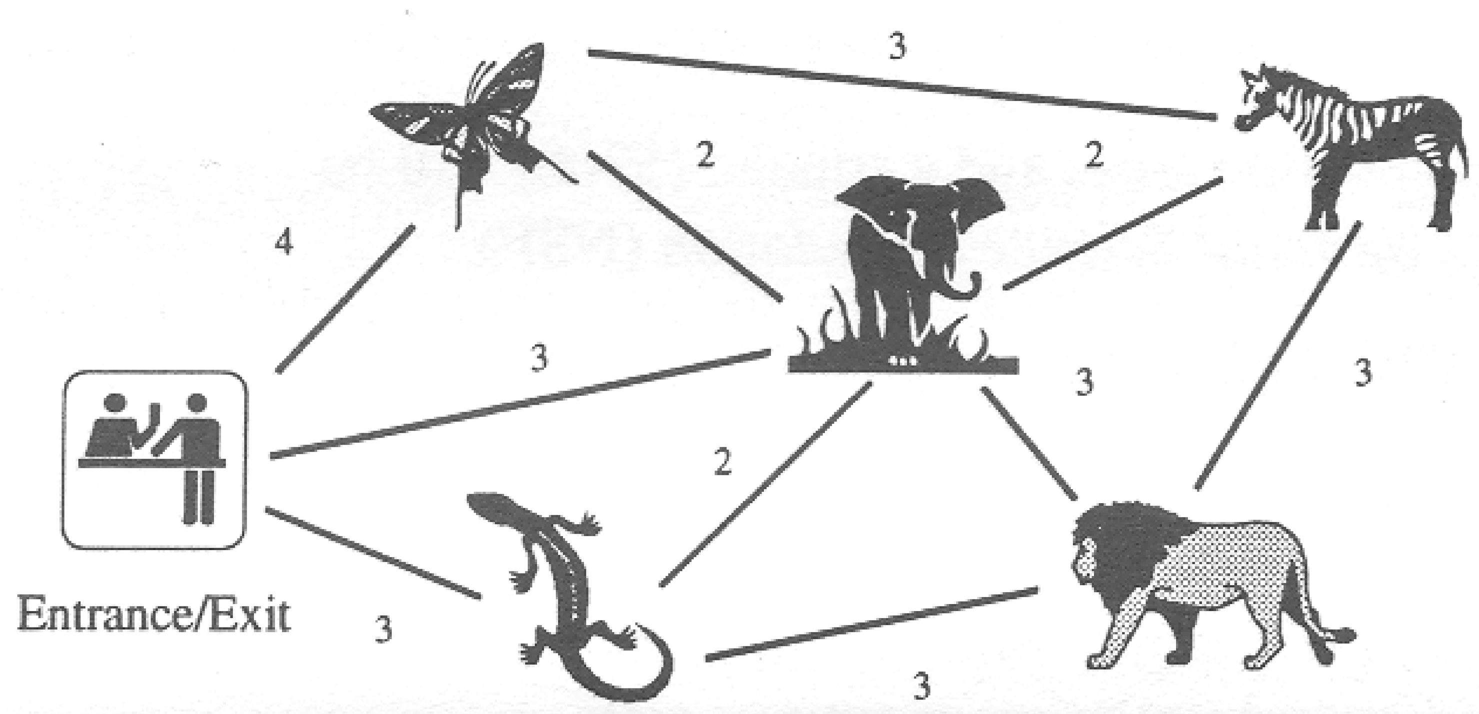
Algorithms *(Continued from page 4)*

cries of "No way!" when the students saw on the overhead that it would take a computer over 19 million years to solve a TSP with 25 cities by exhaustive search. This led to a discussion of saving time and money by using algorithms that could generate a close approximation of the optimal answer. We discussed how researchers at Bell Laboratories have studied this type of problem for years and have come up with various algorithms that are up to 98% accurate and very fast. (See [1], pp. 38-43.) The unit ended with a take-home quiz.

Overall the unit was a success. To improve it, I would give the students another day on TSP problems and a fifth day for making up their own problems.



A Traveling Salesperson Problem
Judy and Jim want to see all the animals at the zoo.
Each path is labeled with the number of minutes it takes to walk it.
What's the shortest circuit they can take? [5]

As I had hoped, the students clearly did not "need" the algorithms to solve the problems—they came up with their own! Isn't that what math is all about?! What they *do* need is the experience of discovering their own methods for solving problems, to prove to themselves that they really are mathematicians, and don't need to be told what to do. They need to be exposed eventually to the "classic" algorithms, because they are useful and are part of the vocabulary of mathematics, but they don't need them right away.

I am still hoping to get the students' enthusiasm and confidence to permeate into what they perceive as "real math." And I am still trying to get them to see that the discrete units *are* real mathematics—wish me luck! ❖

References

[1] COMAP, *For All Practical Purposes*, 3rd ed., W.H. Freeman, NY, 1994.

[2] Roberts, Fred, *Applied Combinatorics*, Prentice Hall, Englewood Cliffs, NJ, 1984.

[3] Problem by Joe Rosenstein, from a story by Mike Fellows.

[4] From Cozzens, Margaret B. and Richard Porter, *Problem Solving Using Graphs*, HiMAP Module #6, COMAP, Arlington, MA, 1987.

[5] Based on a problem by Alistair Carr and Susan Picker.

What's so Special . . .? *(Continued from page 5)*

discrete mathematics topics.

(2) In the October 1994 *Mathematics Teacher* (p. 488) is a letter entitled "birthday converse", from John Koker. You might enjoy figuring out how his letter is related to this problem.

Proving that Kaprekar's Game Works

Here is a sketch of a proof that playing Kaprekar's game (see p. 5) on any four-digit number that is not a multiple of 1111 will yield 6174 in at most seven steps.

To warm up, start by playing Kaprekar's game with two-digit numbers. For example: $61-16=45$, $54-45=09$, $90-09=81$, $81-18=63$, $63-36=27$, $72-27=45$. This time, instead of being stuck at one number, we're stuck in a loop ($45 \to 09 \to 81 \to 63 \to 27 \to 45$).

In the example, you also notice that once you get a difference that is a multiple of 9, after at most one more step you are either stuck at 0 or in this loop. (One step is needed to get into the loop from 18, 36, 54, 72, or 90.) However, after one step of Kaprekar's game, you will *always* get a multiple of 9; the reason is that, given a two-digit number with digits **a** and **b**,

$$10\mathbf{a}+\mathbf{b}-(10\mathbf{b}+\mathbf{a})=(10-1)\mathbf{a}-(10-1)\mathbf{b}=9(\mathbf{a}-\mathbf{b}).$$

Since $\mathbf{a} \geq \mathbf{b}$, $\mathbf{a}-\mathbf{b}$ is a whole number between 0 and 9. Thus, for any two-digit number, after at most two steps of Kaprekar's game, if you're not at the fixed point 0 (which occurs only for multiples of 11), then you are stuck in the "9-loop" of length five.

Now let's look at four-digit numbers, using "**abcd**" as a shorthand for $1000\mathbf{a}+100\mathbf{b}+10\mathbf{c}+\mathbf{d}$. This time you find that (after doing a bit of algebra),

$$\mathbf{abcd}-\mathbf{dcba}=999(\mathbf{a}-\mathbf{d})+90(\mathbf{b}-\mathbf{c}).$$

For example,

$$7641-1467=(999*6)+(90*2)=6174.$$

There are at most ten possibilites for $\mathbf{a}-\mathbf{d}$ and $\mathbf{b}-\mathbf{c}$, so you need to check at most $10*10=100$ numbers to check whether you always reach 6174 in at most 7 steps. If you start to fill in a table of these 100 numbers, you will find that there is a lot of symmetry, and that you really only have to check 30 numbers.

Unfortunately, this "brute-force" approach doesn't really explain "why" 6174 is the only fixed point; better anwers are welcome. Also, I'll let you find out for yourself what happens with three-digit numbers. I don't know what happens for numbers with five or more digits; if you or your class find any results on these, please write to us. —Editor